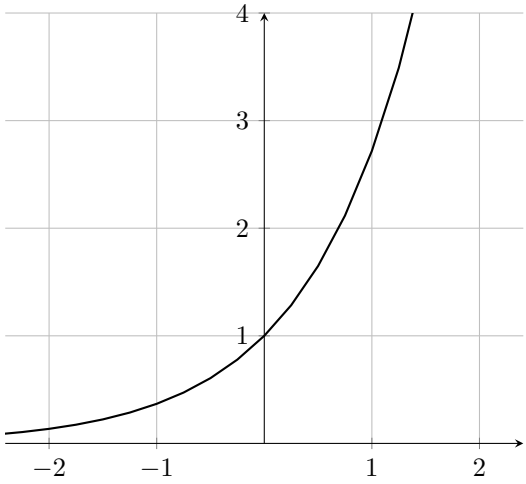
Ex 1 Throughout this problem, we'll use techniques that we already know to create successively better approximations for the function $f(x) = e^x$.

(a) Come up with the constant function (also known as a degree 0 polynomial), $g_0(x)$, that passes through the same point as $f(x)$ at $x = 0$. Graph $g_0(x)$ on the following axes.

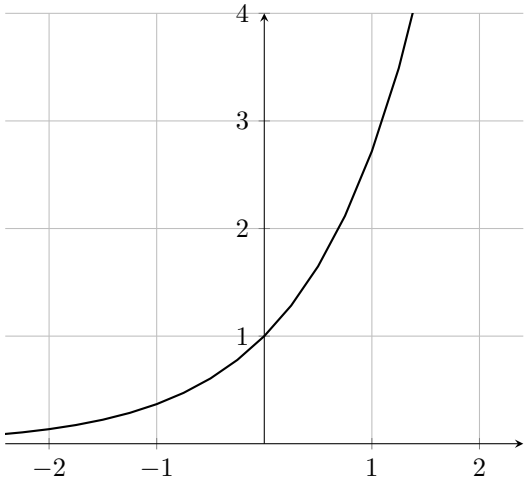
i. Graph $g_0(x)$ on the following axes



ii. Is it possible for $g_0(x)$ to have the same slope as $f(x)$ at $x = 0$? Why or why not?

(b) Come up with the linear function (also known as a degree 1 polynomial), $g_1(x)$, that passes through the same point *and* has the same slope as $f(x)$ at $x = 0$ (this is the tangent line!).

i. Graph $g_1(x)$ on the following axes.

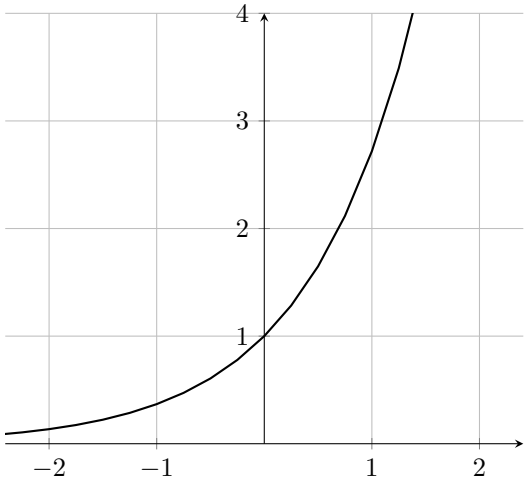


ii. Is it possible for $g_1(x)$ to have the same second derivative as $f(x)$ at $x = 0$? Why or why not?

iii. How does the constant term of $g_1(x)$ compare to the constant term of $g_0(x)$?

(c) Come up with the quadratic function (a degree 2 polynomial), $g_2(x)$, that passes through the same point, has the same slope (first derivative), and the same concavity (second derivative) as $f(x)$ at $x = 0$.

i. Graph $g_2(x)$ on the following axes.

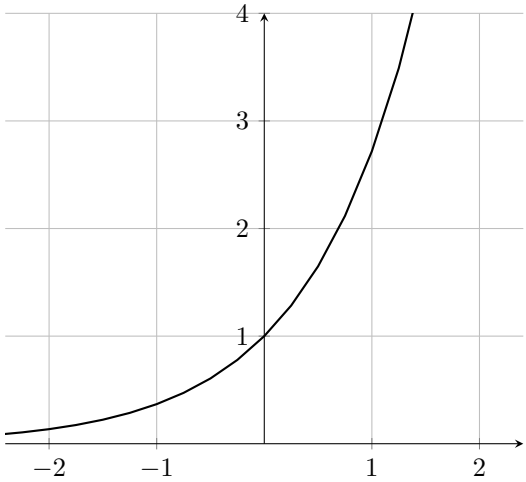


ii. Is it possible for $g_2(x)$ to have the same third derivative as $f(x)$ at $x = 0$?

iii. How do the constant and degree 1 terms of $g_2(x)$ compare to the constant and degree 1 terms of $g_1(x)$?

(d) Come up with the cubic (degree 3) polynomial, $g_3(x)$, that passes through the same point, has the same first derivative, second derivative, and third derivative as $f(x)$ at $x = 0$.

i. Graph $g_3(x)$ on the following axes.



ii. Is it possible for $g_3(x)$ to have the same fourth derivative as $f(x)$ at $x = 0$?

iii. How do the constant, degree 1, and degree 2 terms of $g_3(x)$ compare to the corresponding terms of $g_2(x)$?

(e) Come up with the degree n polynomial, $g_n(x)$, that passes through the same point, has the same first derivative, second derivative,...,nth derivative as $f(x)$ at $x = 0$. The following notation might be helpful: $n! = n \cdot (n-1) \cdot (n-2) \cdots 2 \cdot 1$.